Antenna Temperature Calibration Equations for Radio-SkyPipe

Dave Typinski, July, 2017

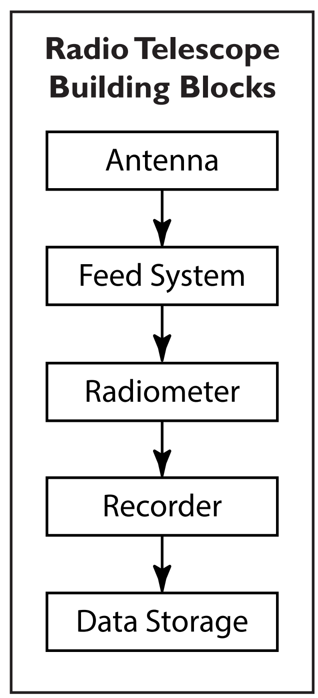
Data generated by a radio telescope must be calibrated if it is to be scientifically useful. To achieve this, the radio telescope's response must be characterized and calibrated. This article provides some background material and describes several methods to calibrate Radio-SkyPipe (RSP) data in terms of antenna temperature (T_{ant}). The relevant calibration equations for each method are discussed and real-world examples using Radio Jove receivers are provided.

The (Imperfect) Parts of a Radio Telescope

It is useful to look at the various parts of a radio telescope in terms of their *response*. A device's response is simply a measure of its output in relation to its input. In radio electronics, we often desire a *linear* response; that is, what comes out reflects what goes in, without distortion. With real-world components, responses are never perfectly linear.

Two kinds of response are important to a radio telescope: amplitude response and frequency response. Frequency response describes how the telescope's output amplitude changes as frequency changes when provided with a constant input amplitude. A nice flat frequency response is important for wide band instruments like spectrographs. Amplitude response describes how a telescope's output amplitude changes as the input amplitude changes given a constant radio frequency. A distortion-free amplitude response is important to all radio telescope receivers and is of primary concern for single-frequency receivers. This article is concerned with amplitude response.

A radio telescope can be though of as a collection of separate building blocks, most of which have an effect on the telescope's response.



Antennas have responses that vary with frequency. Narrow band antennas can be made to be very efficient, but "wide band" antennas used to observe a wide range of frequencies – say, 15 to 30 MHz – will usually have differences in response with changes in frequency.

Feed systems consist of everything between the antenna element(s) and the radiometer input. This includes coax cable feed lines, power combiners, pre-amplifiers, multicouplers, hybrid rings, and switching relays. Coax cable attenuates a radio signal to a greater degree as the radio frequency increases. The other components may also have different attenuations or amplifications at different frequencies.

The **radiometer**, basically a radio receiver without any demodulation function, will not only have a different response at different frequencies, but also an imperfectly linear response with respect to input amplitude. That is, even at a fixed frequency, the response is close to linear only over a certain range of power levels – this linear response region is the receiver's *dynamic range*.

The data **recorder** is generally digital, but still contains real-world parts that may contribute to inaccuracies in the radio telescope's response.

The **data storag**e device is usually a computer hard drive or solid state memory. This is one component that does not introduce errors into the recorded data.

These imperfect parts work together to measure and record radio noise amplitude. However, since they're imperfect, the resulting measurements are not perfect.

This is where calibration comes in. Calibration takes an imperfect response and converts it into something closer to ideal. A telescope's response will never be perfect even with strident calibration; but, that's okay. We don't require perfection. We require only that the telescope's response is reasonably accurate.

But why do we even need to worry about this? Is uncalibrated data useful? Uncalibrated data will let you know that the telescope is turned on and that the antenna is working to some degree. Beyond that, uncalibrated data has little value. One simply cannot do very much with it.

Bandwidth Limited Noise - or How a Radio Telescope Works

So far, all cosmic radio signals detected on Earth consist of radio frequency (RF) noise. If we make a plot of radio noise over time, we get something like Fig 1. This presents us with a problem: since noise is random, how can we measure it? If we are to do science with our radio astronomy observations, we have to be able to quantify this noise. We have to quantify it because science does not allow us to compare a "pretty strong" signal to a "kind of weak" signal. We must use mathematics, which means we have to assign a number (quantify) the RF noise we observe. We must measure it.

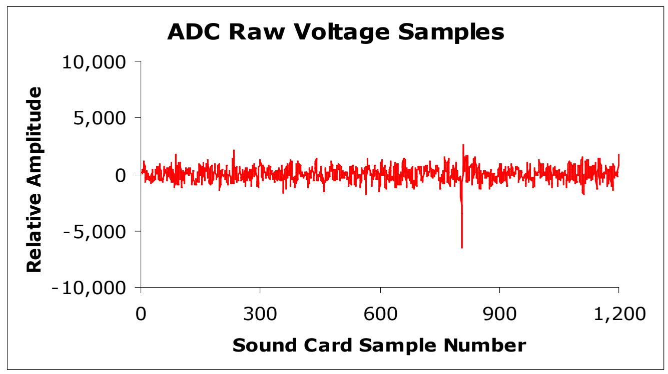


Figure 1 – An example of radio noise.

But how do we measure and quantify something that is completely random by its very nature?

There is one aspect of cosmic radio noise that is easily quantifiable: its power. The power contained in cosmic radio noise is no different than the power contained in any other radio wave. The noise itself, however, is quite different from other radio waves. Most human-generated radio waves are narrow bandwidth, modulated sine waves. Cosmic noise is most often very wide bandwidth radio noise – that is, there is no sine wave carrier signal. It is just pure noise.

The power contained in radio noise – called the *noise power* – may be measured with a device called a *total power radiometer* or just *radiometer*. A radiometer is an instrument not much different from an AC power meter except that it operates at radio frequencies.

All radio telescopes – from Jansky's merry-go-round to the GBT and the VLA and everything in-between, including the Radio Jove telescope – have at their heart a radiometer. Some operate only at one radio frequency with a narrow bandwidth while others operate on many frequencies with wide bandwidths concurrently. Regardless of the details, they all do the same thing: they measure and record the voltage present at an antenna's feed point. This is what radio telescopes *do*. In fact, this is *all* that radio telescopes do.

Despite the simplicity of the concept, the engineering can be complex. There is usually some amplification, filtering, and one or more stages of frequency conversion between the antenna feed point and the output of the radio telescope; but, it's all in an effort to accurately measure the voltage developed at the antenna feed point.

So, say we have some cosmic radio source – the Sun, Jupiter, Cassiopeia A, the Milky Way galaxy, whatever – spewing forth RF noise. This noise impinges upon an antenna and in so doing creates a tiny, randomly-fluctuating voltage across the antenna feed point terminals. We connect a radio receiver to the antenna feed point to filter and amplify the tiny voltages. We add some method of measuring and recording these voltages and *voilà*, we have a radio telescope.

But from whence comes noise power? Voltage is voltage and power is power, right? And if we're measuring voltages, how do we get to power?

And besides, if the noise voltage is *random*, then how come it doesn't simply average to zero?

Well, it *does* average to zero – a look at Fig 1 makes that obvious. But that's not how to measure the power in a radio wave – or any other wave for that matter. For example, if you averaged a bunch of instantaneous voltage readings from the AC mains in your house, the average would be zero since the voltage oscillates between –160 and +160 volts 60 times per second. Obviously the average voltage says nothing about the power available.

We know from Ohm's law that power is equal to voltage squared divided by resistance; that is:

$$P = V^2/R.$$

We can measure the voltage many times, very rapidly, then square those readings (Fig 2), then average them (Fig 3), then divide by R, to obtain power.

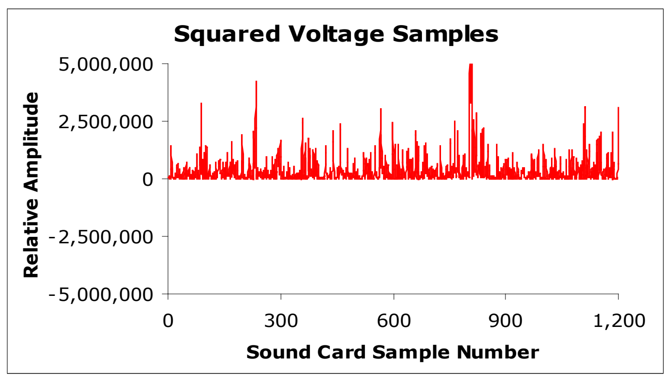


Figure 2 – The same voltages from Fig 1, but squared.

Taking the AC mains as our example again, we know the voltage oscillates at 60 Hz. If we measure this voltage many times per second (say, 150), then square all the voltage measurements, then take the average, then take the square root, we will get about 115 volts. This voltage is known as *RMS voltage*, for root-mean-square – that is, taking the root of the mean of the squares. We can leave off the square root for power calculations, since the equation for power requires V^2, not V.

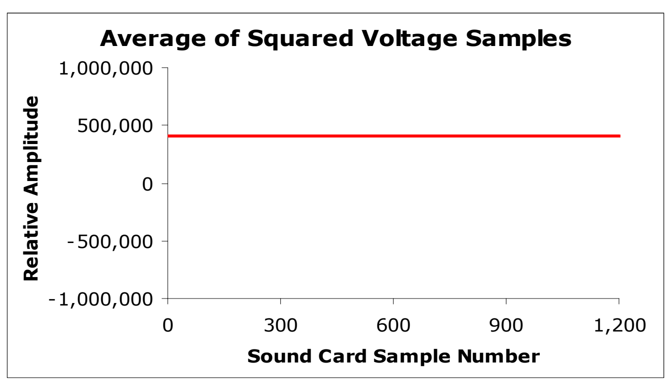


Figure 3 – The squared voltages from Fig 2, now averaged.

But what is R? This is something we must calculate.

For example, If we know an incandescent light bulb uses 60 watts, then:

$$R = (115\ V_{rms})^2 / 60 \text{ watts} = 220\ \Omega$$

Note that this is the resistance of the light bulb's filament at operating temperature, not cold.

We can perform the same calculation for our radio telescope receivers to find R so we can calculate the power of cosmic radio noise. Instead of a light bulb, we use a radio noise source.

If we have a calibration noise source of known output, and we measure V many times, rapidly, we can solve the power equation for R. Once we know R, we can remove the noise source and replace it with an antenna, then calculate the power of the received radio noise.

But what is the known output power of the calibration noise source? Noise generators don't come labeled in watts, but rather in units of kelvin – a *noise temperature*.

Great, so what's noise temperature? This is where an equation for noise power comes in, namely

$$P = kTB$$

where P is power in watts, T is noise temperature in kelvin, B is the bandwidth in hertz over which the noise is being measured, and k is the Boltzmann constant which is equal to 1.38×10^{-23} joules per kelvin.

The bandwidth in this equation is the bandwidth over which the receiver measuring the noise power is sensitive. Radio receivers may tune over a wide range, but when tuned to a specific radio frequency, they can only "see" radio waves over a narrow part of the RF spectrum centered on the tuned frequency. For example, a receiver might be tuned to 20 MHz and be sensitive to radio waves with frequencies between 19.997 MHz and 20.003 MHz – in this case we would say that the receiver's bandwidth is 6 kHz.

Noisy Resistors

Noise temperature is also the physical temperature that a resistor would posess to make the same noise voltage across its leads. That is, all resistors make a small noise voltage depending on their physical temperature. This is due to the electrons jostling around within the resistor. If we have a 50 ohm resistor across the 50 ohm input of a radiometer, and the resistor is at 450 K (about 350 °F), then a calibrated radiometer would read 450 K.

This is where the term *bandwidth limited noise*, or *band limited noise*, comes from. It is the bandwidth of the receiver that is doing the limiting. If the receiver did not limit the bandwidth, the receiver would be sensitive to RF from DC to daylight and tuning would be pointless.

Let's try the noise power equation, assuming our calibration source has an output noise temperature of 24 kK:

$$\text{Noise Power} = kTB = (1.38\times10^{-23}\ \text{J/K})(24{,}000\ \text{K})(6\ \text{kHz}) = 2.0\times10^{-15}\ \text{W}.$$

We connect the noise source to the radiometer's input. Let's say the radiometer measures 710 nV for the RMS voltage for this amount of noise input. (*How* a radiometer measures this voltage and records it is the topic of the next section.)

We can now use the known noise power in watts and the voltage reported by the radiometer to solve for R:

$$R = V^2 / P = (7.1\times10^{-7}\ \text{V})^2 / 2.0\times10^{-15}\ \text{W} = 252\ \Omega.$$

Which means that we can replace the noise source with an antenna, measure the voltage at the antenna feed point many times, very rapidly, then square those voltages, then average them, then divide by 252 ohms, to obtain power in watts at the antenna feed point.

Note that the calculated 252 Ω is only a constant of proportionality that may not correspond to any real resistance in the physical universe. It is, rather, whatever comes out of the equation. That is, it is whatever is required to make the radiometer's output read correctly for the known input temperature. We used 710 nV as an example. But if the same radiometer instead read 430 nV for the same 24 kK input, then R would be something other than 252 Ω.

Since we know how to calculate the power of radio noise, we could calculate power in watts at the antenna feed point using Ohm's law, then use the noise power equation to calculate noise temperature.

But that isn't necessary. Instead of horsing with watts, we simply calibrate the radiometer in terms of temperature directly. Since power and temperature are related by the Boltzmann constant k and bandwidth B, we can easily combine Ohm's law and the noise power equation to make something very useful. To wit:

We know that $P = V^2/R$. We also know that for band limited noise, $P = kTB$. Which means that

$$T = V^2/(kBR).$$

For example, if we hook up the 24 kK calibration noise source to the receiver, the radio telescope should measure 710 nV and thus report and record a noise temperature of 24 kK:

$$(7.1\times10^{-7}\text{ V})^2 / [\ (1.38\times10^{-23}\text{ J/K})(6\text{ kHz})(252\ \Omega)\ ] = 24\text{ kK}.$$

When the radiometer is connected to the antenna, the calculated noise temperature at the antenna feed point is often called, simply, the *antenna temperature*. It is a noise temperature, specifically the noise temperature present at the antenna feed point. It is often abbreviated as T_{ant}.

The subject of this write up is what to do when the radio telescope invariably reports temperatures other than those it should. That's easy: the telescope must then undergo calibration. In concept, a calibration factor, call it a_{cal}, is added to the equation above to give us:

$$T = (a_{cal})(V^2) / (kBR)$$

In practice, we ignore k, B, and R altogether and simply adjust a_{cal} so the radiometer presents the correct reading. We discuss below several ways to determine what a_{cal} needs to be and how to configure it in RSP – and also what to do when a_{cal} isn't a constant.

So, after all that, what value is there in measuring the noise temperature accurately?

The entire field of radio astronomy – from measuring the variations within the CMBR resulting from density variations present in the early universe that have since turned into galactic superclusters and great voids, to measuring the relative velocities of neutral hydrogen clouds within galaxies to infer the existence of dark matter, to observing Jovian decametric emission that allows us to deduce how fast Jupiter's core is rotating beneath all its clouds, to the observation of pulsed radio emission proving the existence of rapidly spinning stellar cinders with surface gravity so strong that their protons and electrons are squashed together to form a ball of neutrons as heavy as a star but only 15 km in diameter – *all* of this is built on nothing more than accurately measuring a bunch of antenna feed point voltages. How cool is *that*?

None of that would have been possible with uncalibrated data. We need calibrated data in order to analyze our data mathematically. Mathematical analysis is required to perform science. If it isn't math, it's just opinion. That's why accurate measurements are important.

How to Measure Voltages and Find Power in the Real World

So much for the conceptual and mathematical background. Let's put this material into practice and see what comes out of a real radio telescope receiver.

A receiver operating as a total power radiometer (that is, no demodulation) is simply a voltage amplifier, bringing the tiny voltages present at the radio telescope antenna feed point up to a level where they can be measured. A good receiver will amplify a wide range of voltages without distortion. This distortion-free range is called the *dynamic range* of the receiver.

Most radio telescopes today measure voltage by sampling the receiver's output noise waveform very rapidly with an analog-to-digital converter (ADC). For example, the Radio Jove telescope uses the ADCs in a computer's sound card to measure the output voltage of the receiver.

When using this method, care must be exercised such that the sampling rate is *at least* twice the highest frequency component in the waveform. If we are using a receiver with a 3.5 kHz bandwidth (BW), such as the Jove receiver, we would want to take voltage readings at a bare minimum of 7,000 times per second. If we don't, we could miss some of the power contained in the higher frequency components of the waveform being measured and our results would be inaccurate. It is good practice to keep the sample rate to about 2.5 times the receiver BW to allow for the real-world characteristics of the filters in the receiver. The most common sound card sampling rate meeting this requirement for the Jove receiver's 3.5 kHz BW is 12,000 samples per second (also known as 12 kHz in sound-card-speak). So, 12 kHz is the sound card sampling rate most often used with the Jove receiver.

Once we have the voltage samples from the ADC's, they are simply squared and then multiplied by a calibration factor, the result being a noise power in terms of antenna temperature. We use a standard noise source to derive the calibration factor. There are several ways of doing that, discussed in the Calibration Method sections below.

RSP Detection Methods

RSP has two ways of handling voltage samples from a sound card's ADCs that are useful for serving as the T_{ant} measuring and recording end of a radio telescope. They are called "detection methods" within RSP, the name coming from an older, analog method of using the square-law region of a diode's voltage-current response curve as a power detector for very tiny RF signals. RSP does the same thing digitally, no diode required. The RSP detection methods that pertain to radio astronomy are "Power" and "Average." The detection method is configured using the RSP menu item Options → Data Source → Detection method.

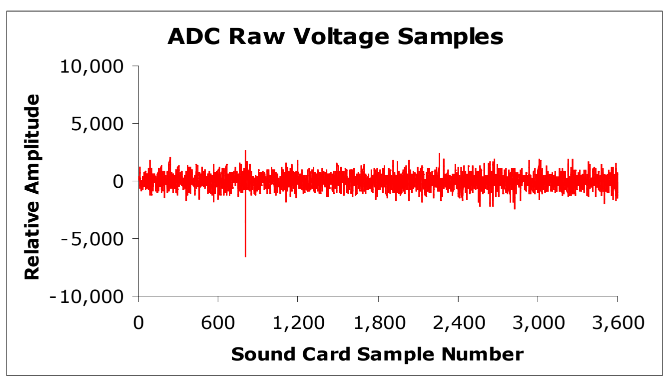


Figure 4 – Raw sound card ADC voltage samples. This is 300 ms of a WAV file recording of the galactic background. Recorded using 12 kHz audio sample rate and 16-bit sample depth. Observation made 7/8/17 at 1841 UTC by the AJ4CO riometer.

In the **average detection method**, RSP averages the absolute values of raw sound card data over the sample period specified. For example, if the RSP sample period is 100 ms and the sound card's sample rate is 12 kHz, then RSP will pull 1,200 samples at a time from the sound card's ADC(s). These samples are usually 16-bit integers (values range from -32,768 to +32,767) and represent the voltage of the audio waveform present on the sound card's input(s). Fig 4 shows sound card sample vales from three consecutive 1,200-sample batches of data.

Note that we don't know the actual voltage, only that the ADC's output is linearly proportional to the waveform's instantaneous voltage. The ADC uses a voltage reference V_{ref}, then divides the voltage range between $-V_{ref}$ and $+V_{ref}$ into 65,536 divisions (assuming a 16-bit sample depth, common for sound cards and WAV files). Half of these divisions are negative, the other half positive. These divisions can be envisioned as buckets. The ADC simply finds the appropriate bucket for the waveform's voltage at time *t* and reports the bucket number to the software collecting the ADC voltage data. If we knew the ADC reference voltage, we could obtain the actual waveform voltage. This is how digital oscilloscopes work. Since we calibrate the telescope against a noise source, we don't need to know the true voltages.

After RSP finds the absolute value of each of the 1,200 samples (Fig 5), RSP find the arithmetic mean – i.e., the average of the 1,200 samples (Fig 6) – and records that average as one data point in the strip chart data file (Fig 7). Using the average detection method, each data point within an SPD file is generated using the equation below, where x_i is one sound card sample and n is the number of sound card samples being averaged (in this example, n = 1,200).

$$\text{Average} = \frac{1}{n}\sum_{i=1}^{n}|x_i|$$

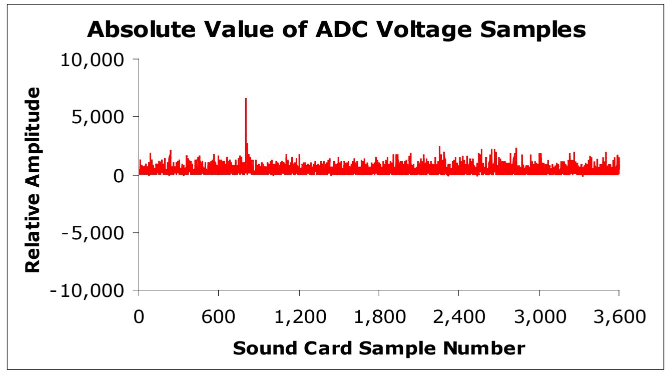


Figure 5 – The data from Fig 4 after finding the absolute value of each data point.

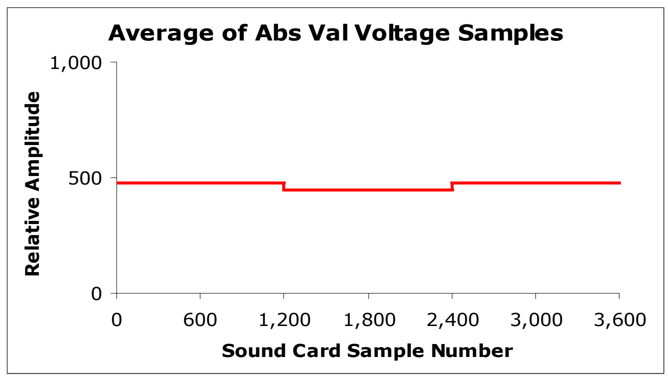


Figure 6 – The data from Fig 5 after averaging over each group of 1,200 samples.

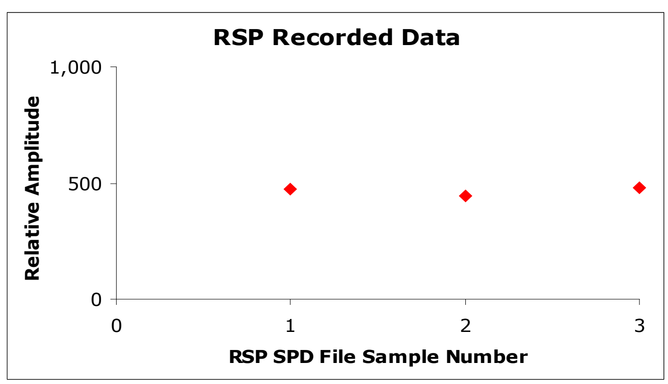


Figure 7 – The averages from Fig 6 as recorded in the SPD data file made by RSP. Each data point represents 100 milliseconds

In the **power detection method**, RSP obtains the same number of samples from the sound card (Fig 4). Since power is proportional to the square of voltage, RSP finds the squares the samples (Fig 8) instead of using absolute values. The squares are then averaged together to find the average power (Fig 9). This average is recorded as one data point in the strip chart data file (Fig 10). Using the power detection method, each data point within an SPD file is generated using the equation below, where x_i is one sound card sample and n is the number of sound card samples being averaged (in this example, n = 1,200).

$$\text{Power} \propto \frac{1}{n}\sum_{i=1}^{n} x_i^2$$

Note the use of the proportionality symbol in the equation for power. More operations are required if we want RSP to record power calibrated in terms of antenna temperature. We must find a scaling coefficient, also known as a constant of proportionality – which is sometimes not-so-constant if we wish to push the telescope to have a wider dynamic range. Adding the scaling coefficient to the data processing flow is precisely what calibration equations do.

Noise Generators / Calibrators

In order to perform a calibration, one must know the output temperature of the noise generator over the center frequency and bandwidth of the receiver.

Common noise generators include the RF-2080 (25 kK fixed), the HP 461 with a step attenuator (1 kK to ~75 MK), and the FBX-100 automatic calibrator (1 kK to 100 MK).

The (uncommon) noise generator used at AJ-4CO Observatory consists of a 440 MK noise source split four ways (6.2 dB loss) followed by a programmable step attenuator (0.56 dB insertion loss) for a maximum temperature at the calibration plane of 440 MK - 6.2 dB -

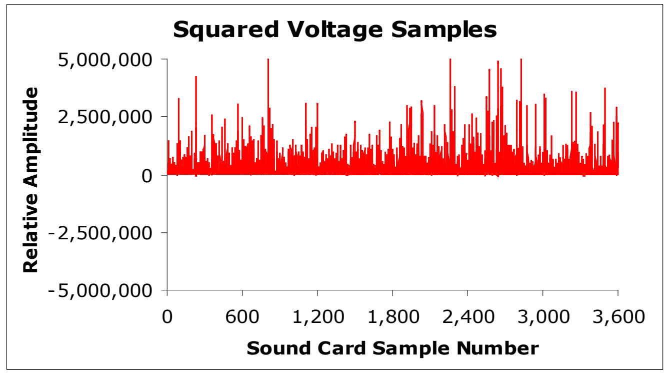


Figure 8 – The data from Fig 4 after finding the square of each data point.

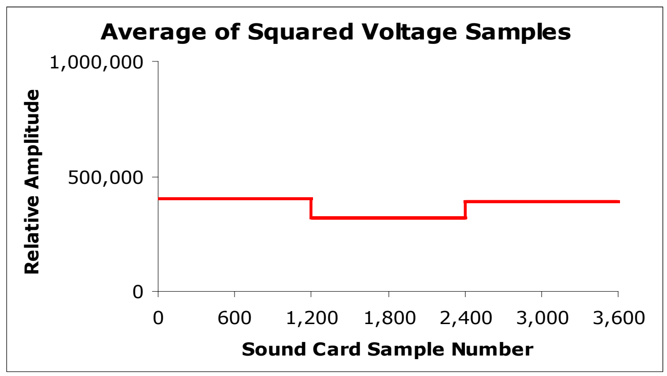


Figure 9 – The data from Fig 8 after averaging over each group of 1,200 samples.

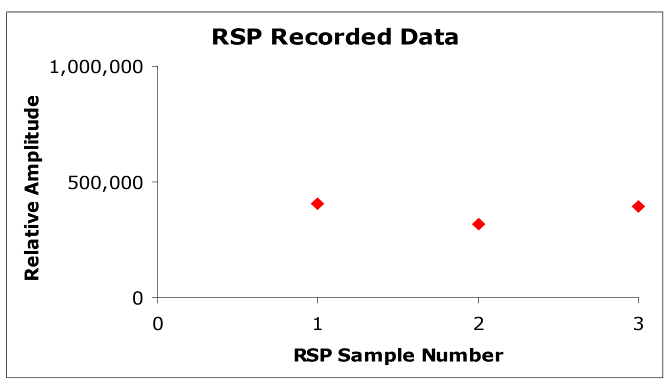


Figure 10 – The averages from Fig 9 as recorded in the SPD data file made by RSP. Each data point represents 100 milliseconds

0.56 dB = 93 MK. This noise source is part of a system that automatically runs step calibrations consisting of 17 temperature steps each 3 dB apart.

All of the output noise temperatures discussed in this article are at 20 MHz.

Feed System Losses

In order to perform a calibration, one must know all the losses in the system between the sky side of the antenna (the antenna itself can be lossy) and the calibration plane (the point at which the calibration noise temperatures exist). In this article, the cal plane lies at the receiver input connector or very close to it.

Losses may include:

- ◊ Feed line insertion loss (around 2.2 dB per 100 feet for Belden RG-58; around 1.5 dB per 100 feet for Belden RG-59)
- ◊ Feed point to feed line impedance mismatch loss (close to zero for Jove dipoles)
- ◊ Balun insertion loss (essentially zero for the Jove dipoles)
- ◊ Impedance matching transformer insertion loss (not applicable to the Jove dipoles)
- ◊ Power combiner insertion loss (usually around 0.2 dB per combiner)
- ◊ Feed line to receiver impedance mismatch loss (0.1 dB for Jove dipoles with 75 Ω feed systems, essentially zero for Jove dipoles with 50 Ω feed systems)
- ◊ Antenna inefficiency (efficiency is roughly 100% for the dipoles in a Jove Array)

In the case of the AJ4CO riometer – a Jove array with reflector wires 7′6″ below the elements feeding an Icom R8500 tuned to 20.1 MHz with the IF output going to a crystal-tuned 10.7 MHz Jove receiver – the losses between the array element feed points and the cal plane are 3.2 dB. The cal plane is at the relay in the automatic calibrator just ahead of the R8500's antenna input connector. The loss in the relay and the jumper from the calibrator to the R8500 is considered to be zero dB.

When the calibration noise source is connected near the receiver, the temperature at the cal plane must be increased by the losses mentioned above to find the equivalent antenna temperature. For example, the AJ4CO Auto Calibrator has a maximum output temperature of 93 MK. However, since there is 3.2 dB of feed line loss between the riometer antenna and the cal plane, it would take an antenna temperature of 93 MK + 3.2 dB = 194 MK at the feed points to equal this temperature at the cal plane.

Therefore, when calibrating, one must know not only the noise source temperature, but also the losses within the feed system so one may calibrate in terms of antenna temperature instead of cal plane temperature.

Calibration Method: Single Step, Single Channel (RSP Calibration Wizard)

During this procedure, a noise generator is placed on the receiver input in place of the antenna feed line. The calibration plane is thus located at the receiver antenna input connector.

The RSP Calibration Wizard, available from the RSP menu item Tools → Calibration Wizard, performs a calibration at one temperature. The Cal Wizard calculates the appropriate scaling coefficient R for an equation of the form $T_{ant} = V^2/R$, where V^2 is the average of the squares of the sound card ADC voltage samples during one RSP sample period. RSP calls the scaling coefficient R the "power detection factor."

When using the Cal Wizard feature, the user must tell RSP the noise generator temperature and the total feed system loss between the array element feed points and the cal plane. The Cal Wizard will then automatically configure RSP to display calibrated antenna temperature based on those inputs.

Figure 11 shows how the uncalibrated output of the AJ4CO riometer compares to known equivalent antenna temperature before running the RSP Calibration Wizard. This was accomplished by running the AJ4CO Automatic Calibrator through its 17 steps and plotting the results.

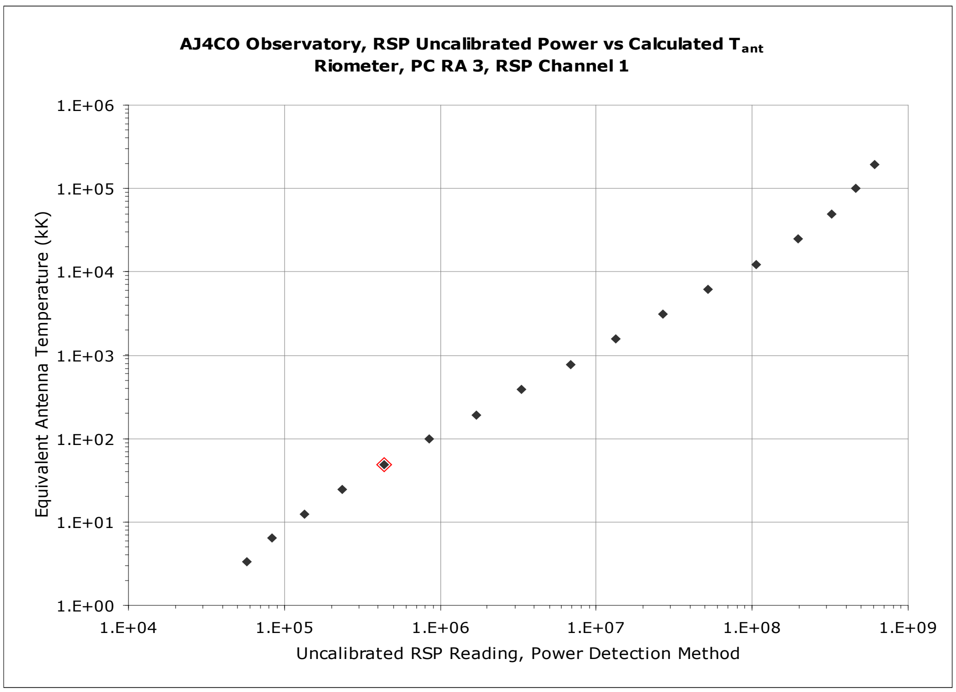


Figure 11 – Uncalibrated power averages to T_{ant}. This data was recorded just prior to the Cal Wizard run. Red dot indicates fixed temperature of subsequent Cal Wizard run.

Note that in Fig 11, as in all such plots throughout the remainder of this article, the known equivalent antenna temperature is on the y-axis and the RSP reading is on the x-axis. This is backwards from the way one might intuitively do it; but, there is a very good reason for this that will become apparent in the sections below on Multiple Step Calibrations in which we generate our own calibration equations in lieu of using the Cal Wizard.

During the Cal Wizard run performed immediately after taking the data in Fig 11, the Cal Wizard adjusted the Sound Card Offset to 0 and the Power Detection Factor to 8710.47.

The first operation the Cal Wizard does is to measure the output of the sound card with the receiver turned off. Some sound cards exhibit a DC offset in their outputs and others do not. This reading is then installed as an offset in RSP to be subtracted from all further sound card data. In this case, the sound card apparently exhibited no DC offset.

The power detection factor *R* is divided into the average of squared sound card voltages by RSP. That is, the power detection factor of 8710.47 is *R* in the equation $T_{ant} = V^2/R$.

In this cal wizard run, RSP was told that the feed line loss was 3.2 dB and that the noise generator temperature was 24 kK (the output of the AJ4CO auto calibrator at its 36 dB attenuation setting). This means the noise gen was providing the equivalent of 24 kK x $10^{0.32}$ = 50 kK antenna temperature to the receiver input connector.

The RSP cal wizard then measures the average sound card ADC power (i.e., squared then averaged ADC voltages). In this case the average of the squared voltage samples was 438,033. RSP then solves the $T_{ant} = V^2/R$ equation for *R*. Taking 438033 / (50 kK) = 8761. The Cal Wizard run came up with a very slightly different number, 8710; this is due to the unavoidable small variations from run to run when the sample periods are less than infinitely long and the noise source and receiver are not perfectly stable.

In this example, the resulting equation that RSP applies to the sound card data to calibrate the strip chart in terms of antenna temperature in kK is:

$$T_{ant}\,(\text{in kK}) = \frac{1}{8710.47\,n}\sum_{i=1}^{n} x_i^2$$

In this instance, x_i is one sound card voltage sample and n = 1,200 (the sound card runs at a 12 kHz sample rate and RSP is configured in this case for 100 ms samples).

To check the accuracy of the Cal Wizard's results, we then run a step calibration using the AJ4CO Auto Calibrator, comparing RSP's reported T_{ant} values to what they should be knowing the calibrator's output at each step. Figure 12 shows the error remaining in the riometer's T_{ant} output after the RSP Calibration Wizard has run through its calibration procedure.

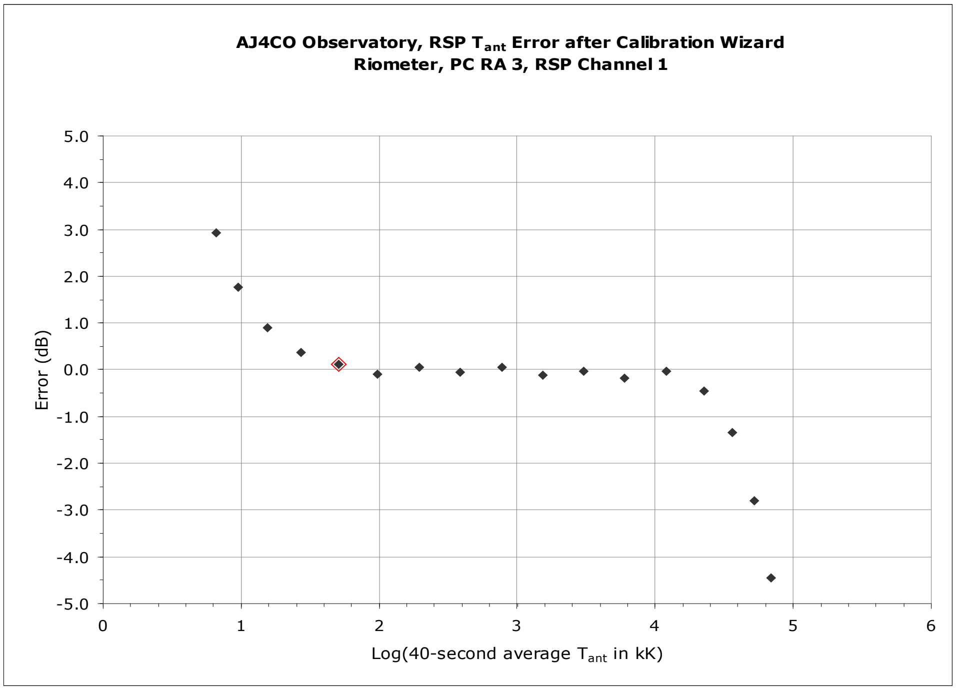


Figure 12 – Error remaining after Calibration Wizard procedure. The red dot represents the fixed noise gen temperature equivalent to 50 kK antenna temperature used during the Cal Wizard run.

The middle of the response curve is nice and flat and very close to zero; but, the dynamic range is only about 25 dB. We see the receiver going into compression at the high end and insensitivity at the low end.

Calibration Method: Single Step and Multiple RSP Channels (Cal Wizard)

We can also run the Calibration Wizard using two receivers, one on each RSP channel. In this example, we use the AJ4CO Jove receivers on the TFD Array's RCP and LCP outputs. The procedure is the same as described above; however, each receiver (RSP channel) must be calibrated separately since the Cal Wizard can only operate on one channel at a time.

The Jove Receivers on the RCP and LCP outputs of the AJ4CO TFD Array have 4.3 dB feed system loss. At 20 MHz, they also have 3.5 dB element inefficiency; this was calculated by comparing the output of a single TFD element to that of a single half-wave dipole assumed to be 100% efficient. This element inefficiency acts just like additional feed line loss. We thus use a total "feed line" loss of 4.3 + 3.5 = 7.8 dB to be entered into RSP's Cal Wizard dialog.

The procedure is the same as for a single channel. We run the Cal Wizard first for one channel, then go back and run it again for the second channel.

There is one interesting difference in the results shown in Fig 13: in this case, the receivers do not go into compression at the maximum temperature of the step cal test. The the low ends, however, are still hampered by insensitivity. This probably indicates more amplification (or less loss) is needed ahead of the receivers, or the receiver AF gain needs to be increased. Fig 14 shows that the balance between the two RSP strip charts is very good except at the low end where each receiver experiences insensitivity slightly differently.

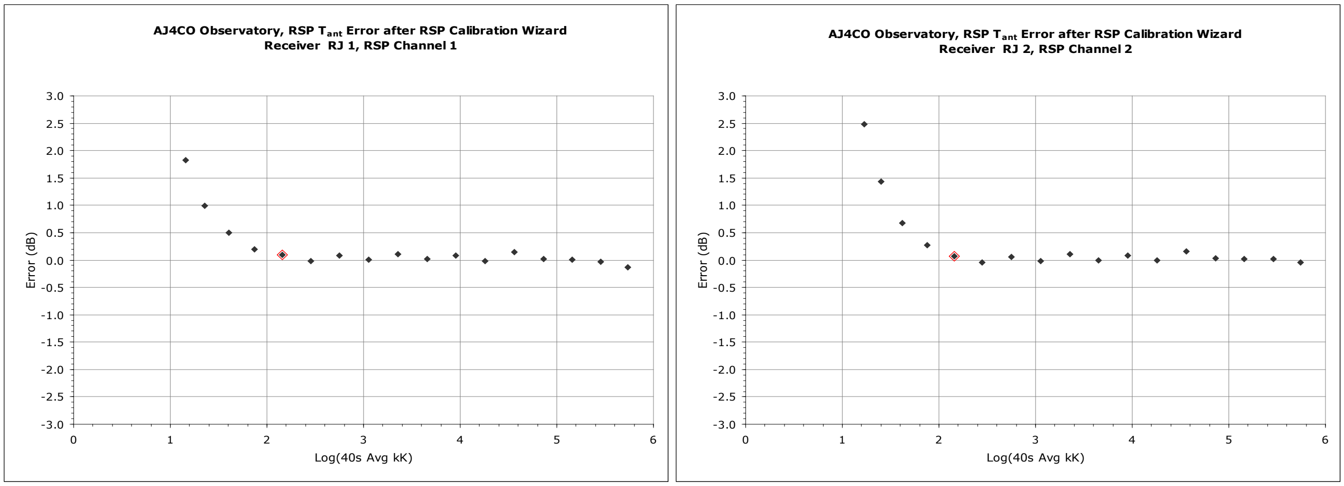


Figure 13a and 13b – Error remaining after Calibration Wizard procedure. The red dots represents the fixed noise temperature equivalent to 142 kK antenna temperature used during the Cal Wizard run – 24 kK at cal plane plus 7.8 dB loss (4.3 feed loss and 3.5 dB element inefficiency) = 142 kK.

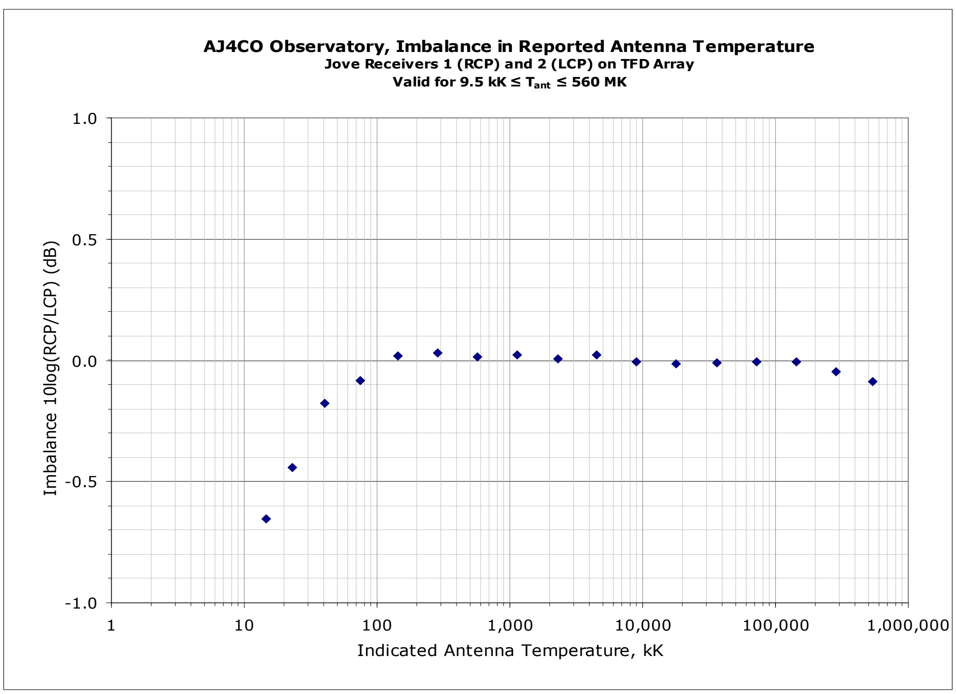


Figure 14 – Imabalance in reported T_{ant} between the two receivers after the Calibration Wizard procedure.

Calibration Method: Multiple Step, Single Channel

This procedure involves using a noise source and performing a series of measurements to arrive at a set of calibration equations that will convert the sound card's ADC voltage averages to calibrated antenna temperatures. We use the calibration of the AJ4CO riometer as our example; however, the procedure applies equally well to any receiver.

Note that we use the Average detection method in RSP for this, not the Power detection method. We do this because we are not sure that the radio telescope perfectly follows a square law response. Using the Average detection method allows us to calculate our own exponent. Note that the error plotted in Figs 12 and 13 after the Cal Wizard assumed an exponent of exactly 2 show that the error is minimized over the receiver's dynamic range. Therefore, an exponent of exactly 2 seems to be correct.

A 17-step calibration is performed using the AJ4CO automatic calibrator. The resulting SPD file is reviewed and each step measured in terms of average ADC voltages (that were suitably absolute-valued by RSP). These measurements are curve-fitted in Excel against the known equivalent antenna temperature at each step to arrive at an exponential equation of the form $T_{ant} = Ax^b$, where x is the average ADC voltage and A and b are determined by a least squares fit. See the AJ4CO Observatory Description linked in the References section for the antenna temperatures. Figure 15 shows the data and a curve fit to the telescope response.

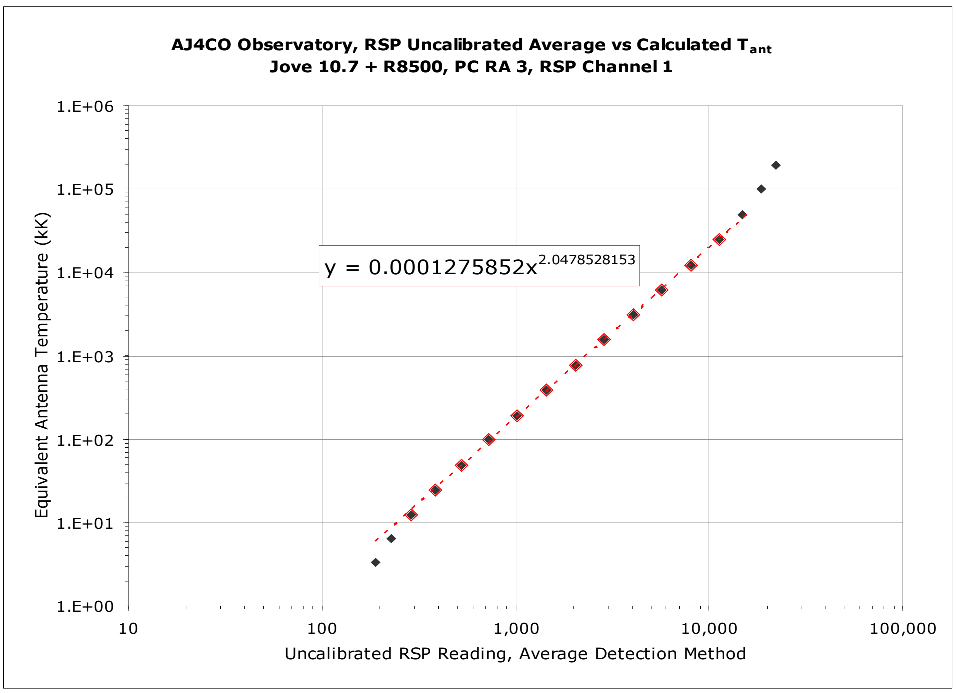


Figure 15 – Equivalent T_{ant} vs. uncalibrated 40-second voltage averages. Red dots indicate data points that were used for curve-fitting. The exponent in the fitted curve's equation is very slightly greater than 2 because the low end data point may be slightly into the insensitive region and the high end data point may be slightly into compression. An exponential always appears as a straight line in log-log space. Thus, the slight compression and insensitivity of the response data points used for curve fitting rotate the fitted curve slightly in the counter-clockwise direction as seen on this plot, which causes the exponent to be slightly greater than 2.

As noted earlier about Fig 11, the known equivalent antenna temperature is on the y-axis and the RSP reading is on the x-axis. This is backwards from the way one might intuitively do it; but, this is a good way to generate a calibration equation using curve fitting. Here's why.

We know what x is at each step since x is simply whatever number RSP spits out for an average amplitude value. In this example, we used 40-second averages.

We know what y should be at each step of the step cal because we know what the noise gen's output is at each step and we know what the 20 MHz feed line loss is, which gives us the equivalent antenna temperature at each step (see the section on Feed System Losses).

So, knowing all the x's and y's, we can make a curve that fits the data points. The equation of this curve is a transfer function that takes an uncalibrated RSP average as its input and provides us with a calibrated output. We call it a calibration equation.

In effect, we're asking, "if the RSP output is this x-axis value, what does that correspond to in terms of calibrated antenna temperature?" Curve fitting gives us a function f such that x is the uncalibrated RSP output (shown on the x-axis) in arbitrary squared (power detection method) or linear (average detection method) voltage units and $f(x)$ is the calibrated antenna temperature (shown on the y-axis) corresponding to x. The function f may be an exponential ($y = Ax^b$) or a polynomial ($y = a_1x^n + a_2x^{n-1} ... + a_nx + c$). Other functions may be useful as well.

The equation fitted by Excel is an exponential, $T_{ant} = 0.000127585x^{2.0478528}$ where x represents the average sound card ADC voltage over one RSP sample period. This equation is then entered into RSP's Equations dialog available in the Options → Data Source → Equations menu item as shown in Figure 16. This converts the ADC voltage from the sound card to T_{ant}. RSP thus records T_{ant} in the data file (SPD file). Note that the inverse of the constant of proportionality (1/0.000127585 = 7837.91) is somewhat near to the power detection factor (8710.47) calculated by RSP's Cal Wizard – but not quite the same. This is mostly due to the fact that the exponent in the fitted curve is not exactly equal to 2. A small amount of difference from the Cal Wizard number is due to random variations when measuring noise over a non-infinite time period.

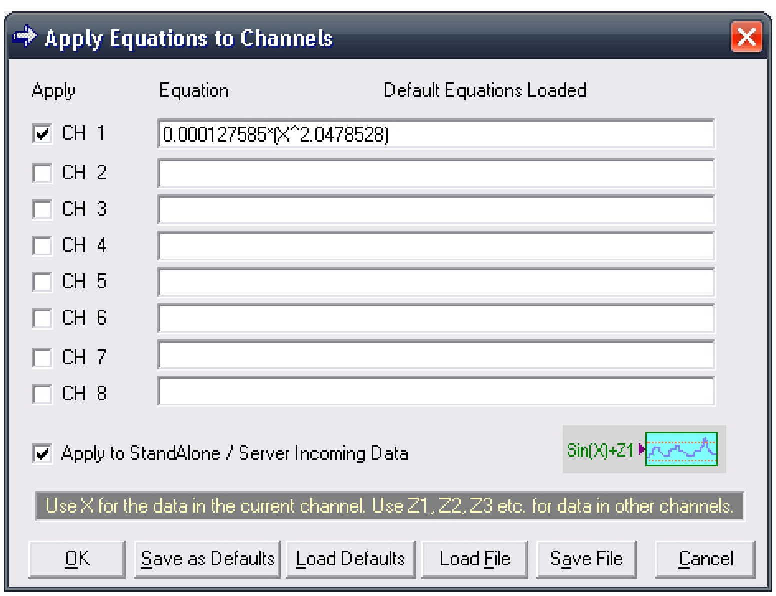


Figure 16 – RSP equations editor window.

This equation is only the first cut at calibrating the receiver's response. With this calibration equation, the response becomes calibrated in terms of T_{ant}, but the dynamic range is still too narrow: the low end is insensitive due to receiver noise figure and the high end shows one or more of the IF amplifiers going into compression as shown in Fig 17.

After the first calibration equation is in place in RSP, a new set of measurements is obtained to arrive at the remaining error in terms of dB at the base-10 logarithm of each temperature reported as shown in Fig 17. Using the log of the temperature makes further curve fitting much easier.

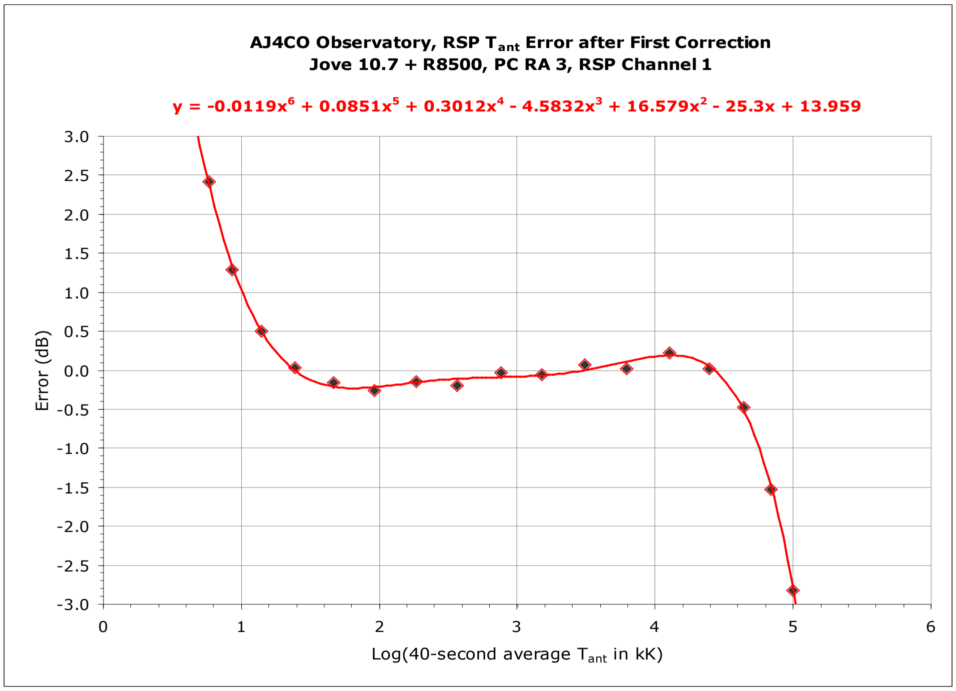


Figure 17 – Error remaining after first calibration equation. Note the positive slope in the linear reagion – this results from the exponent in the first calibration equation being slightly greater than 2. It would be flat if the exponent were 2 exactly, as shown in Fig 12.

Excel is again used for curve fitting, this time to find a polynomial that minimizes the error in reported T_{ant} as shown in red in Fig 17. This second calibration equation is applied to the result from the exponential equation discussed above. Note that the polynomial is operating on the temperatures in the logarithmic domain instead of the linear domain. The intent is to add or subtract the measured error in terms of dB from the result of the first equation.

The RSP equation code is show below. Ignore the line wrap, it is one long line.

```
A=0.00012758*(X^2.0478528);B=log10(A);C=-0.0119*B^6+0.0851*B^5+0.
3012*B^4-4.5832*B^3+16.579*B^2-25.3*B+13.959;A*10^(-C/10)
```

This equation shown above is then entered into the RSP CH 1 equation field as shown in Fig 18.

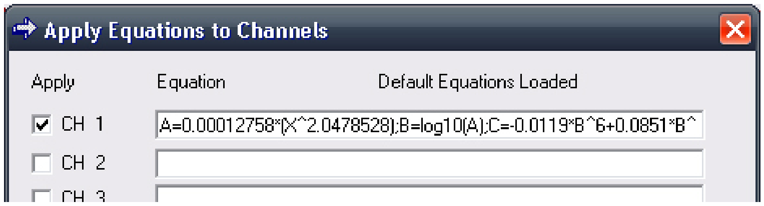


Figure 18 – First and second claibration equations entered into RSP.

A new set of measurements are then performed to arrive at the error remaining after the first and second calibration equations are entered into RSP. This is about as flat as one can get, ±0.15 dB over a ~50 dB dynamic range as shown in Fig 19.

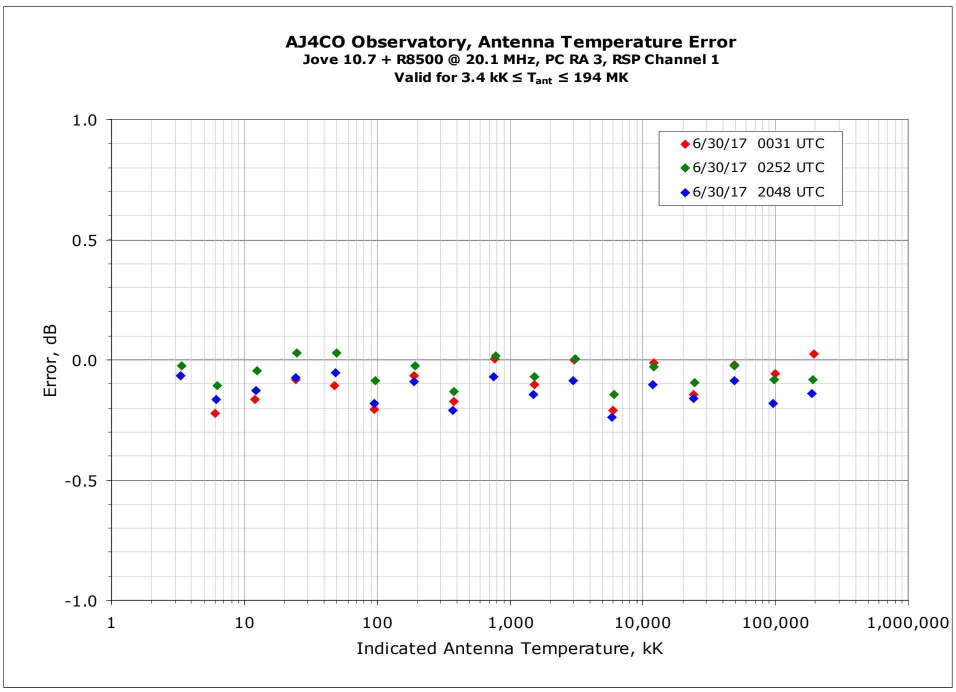


Figure 19 – Error remaining in RSP's indicated T_{ant}.

However, while the response is relatively flat, it is also shifted about 0.1 dB too low on average. To compensate for this, we multiply the result of the second calibration equation by $10^{0.01}$ to increase the recorded temperature by 0.1 dB. We arrive at the final RSP calibration equation below.

```
A=0.00012758*(X^2.0478528);B=log10(A);C=-0.0119*B^6+0.0851*B^5+0.30
12*B^4-4.5832*B^3+16.579*B^2-25.3*B+13.959;A*(10^(-C/10))*(10^0.01)
```

The error measured after this equation is shown in Figure 20.

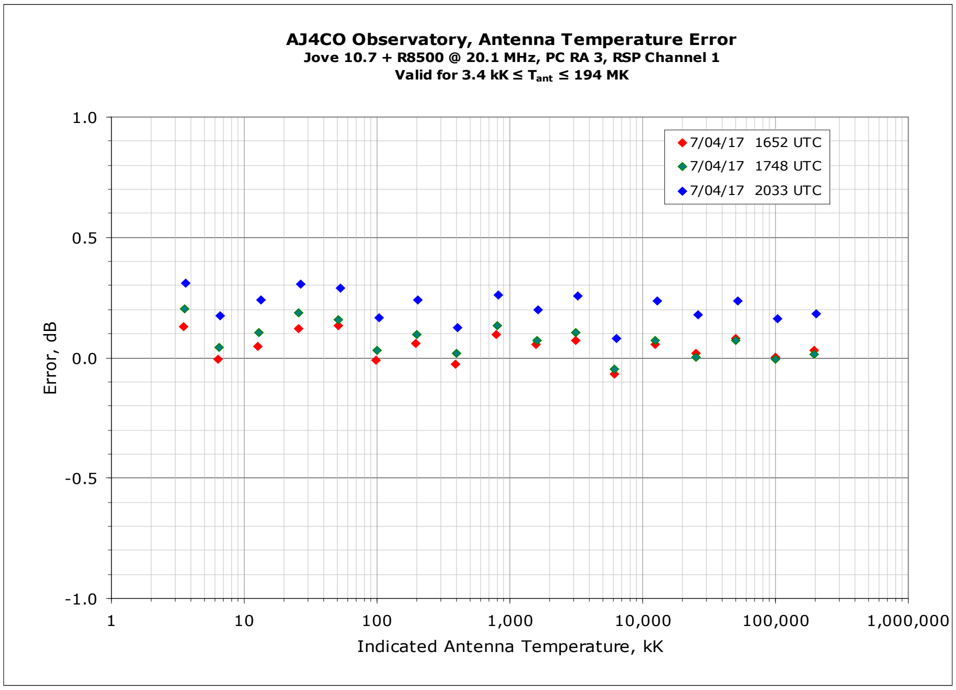


Figure 20 – Error remaining in RSP's indicated T_{ant}.

Note that the variation in error from run to run within the same day is around ±0.15 dB. Comparing Figs 19 and 20 (and acknowledging the addition of 0.1 dB in Fig 20) also shows that the variation over a few days is around 0.2 dB. Trying to get the reported temperature accurate (not just flat) to within 0.15 dB was overly optimistic. The correction provided by the last equation (multiplying by $10^{0.01}$) was washed away by random variation – we went from an average of 0.1 dB too cold to an average of about 0.15 dB too hot. We believe this systematic variation is due to a combination of a) long period variation in the noise generator output temperature and b) variation over time in the receiver's response. The data shows that the best we can do with present instrumentation is around ±0.25 dB accuracy over a few days over a roughly 50 dB dynamic range. Determination of the causes of this variation, such as a correlation between the day-to-day variations in reported T_{ant} and variations in instrument temperature with heating and air conditioning cycling, is left for future investigation.

Calibration Method: Multiple Step and Multiple RSP Channels

The multi-step procedure described above can be extended to RSP setups recording more than one receiver output at a time. We use the calibration of two Jove Receivers on the RCP and LCP outputs of the AJ4CO TFD Array (4.3 dB feed system loss + 3.5 dB element inefficiency loss at 20 MHz) as our example; however, the procedure applies equally well to any pair of receivers.

Recording the initial step calibration proceeds much as shown above, but with one interesting difference: in this case as shown in Fig 21, the receivers do not go into compression and the low end is hampered much more by receiver noise figure than the riometer receiver. This probably indicates more amplification is needed ahead of the receiver or the receiver AF gain needs to be increased. However, since we are crafting our own calibration equations, we can adjust for this using nothing more than calibration equations in RSP.

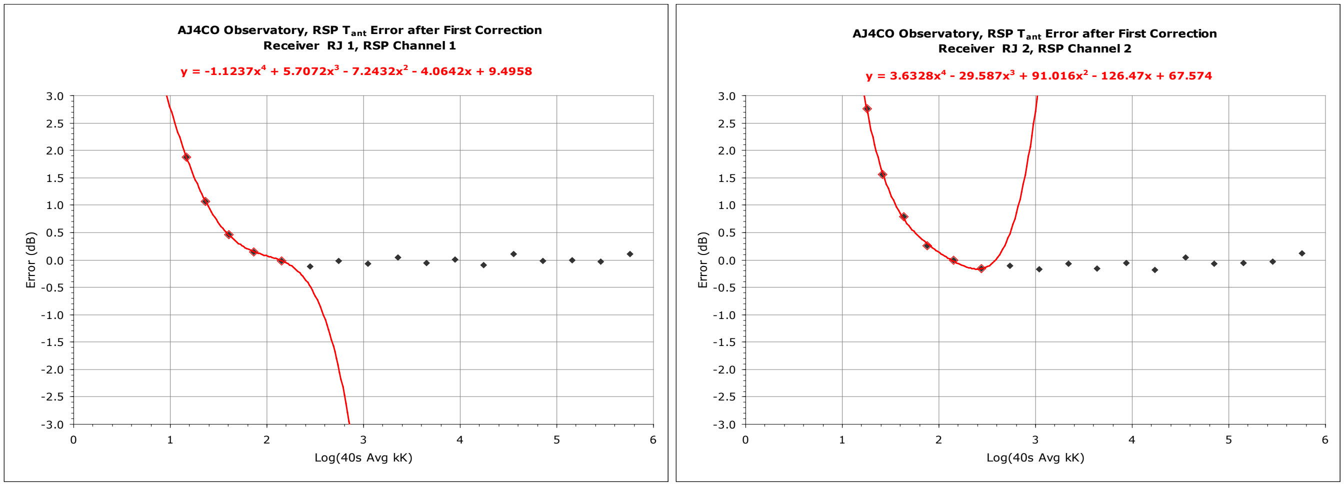


Figure 21a (left) and 21b (right) – Similar to Fig 17, the error remaining after first calibration equation.

After the first calibration equations (the exponentials) were generated and applied in RSP, the responses were flat enough except at the low end. Polynomials in the log domain were fitted to the data. Since the responses above the low ends do not require further calibration, this time the polynomials are only applied below the temperatures at which the polynomials first cross zero. This also means polynomials of relatively low order can be used since only a small portion of each fitted curve must correspond to the data. That is, the tails of the polynomials in Fig 21 that are flying off into infinity won't be used, so we can ignore them. We care in these cases that only that the polynomials fit well to the first 5 or 6 data points.

The zeros of the polynomials are found using Excel's Goal Seek function or by manual iteration. Either way, care must be taken with the initial guess lest one arrive at the wrong zero for polynomials that have more than one zero.

The resulting polynomials are shown in red in Fig 21. The zeros are 134.77 for Fig 21a and 129.65 for Fig 21b.

The RSP equation parser includes an If-Then-Else structure that can be used to accomplish the selective application of the polynomials. The resulting RSP equations are as follows.

For RSP Channel 1 (TFD Array RCP):

```
A=0.00341348*(X^2.0111827);B=log10(A);C=-1.1237*B^4+5.7072*B^3-7.24
32*B^2-4.0642*B+9.4958;IF{[A]<[134.77]};THEN{A*10^(-C/10)};ELSE{A}
```

For RSP Channel 2 (TFD Array LCP):

```
A=0.00361555*(X^2.0158154);B=log10(A);C=3.6328*B^4-29.587*B^3+91.01
6*B^2-126.47*B+67.574;IF{[A]<[129.65]};THEN{A*10^(-C/10)};ELSE{A}
```

After these equations are installed, the remaining errors are measured and are shown in Fig 22.

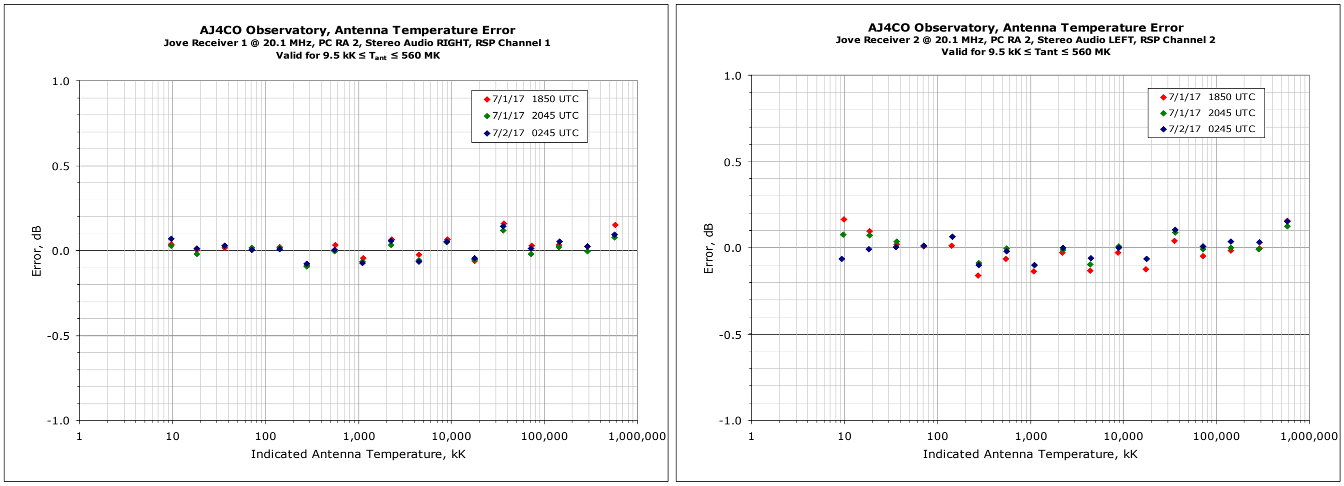


Figure 22a (left) and 22b (right) – Similar to Fig 19, the error remaining in the responses of the RCP and LCP Jove receivers after second calibration equations.

Given the experience with calibrating the riometer's RSP response, it is doubtful the RSP responses of these two receivers can be meaningfully improved.

Another useful thing to know when comparing two receivers on the same antenna is how close the receiver responses are to each other as shown in Fig 23.

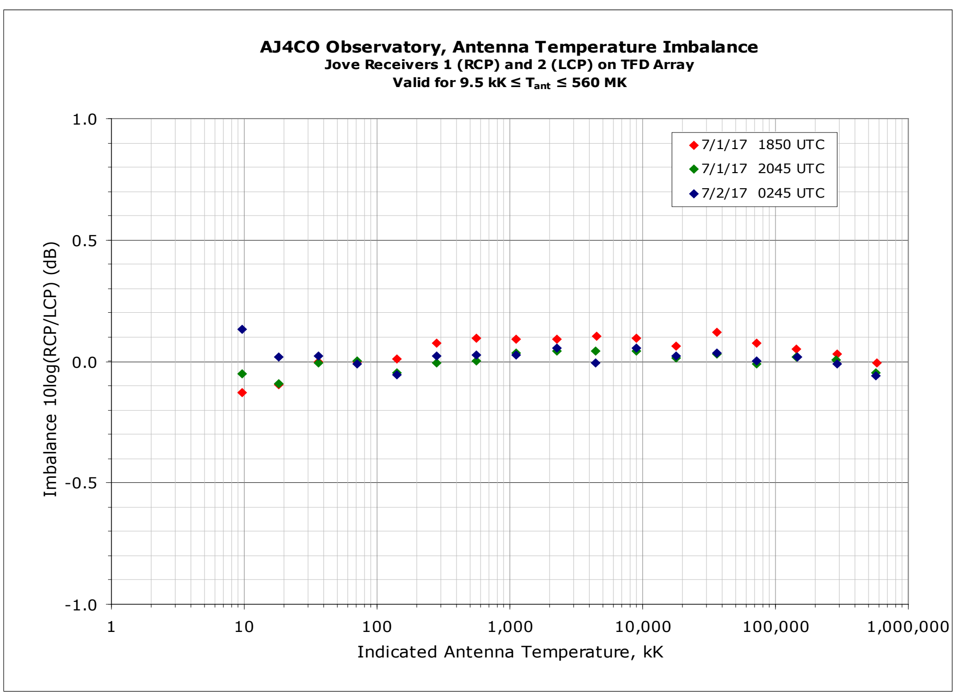


Figure 23 – Imabalance in reported T_{ant} between the two receivers.

A Word on Raw Data

In all the examples discussed above, the raw sound card voltage averages are not recorded.

For the most part, this is okay. However, if the researcher ever wants to re-calibrate the data, having access to the original sound card averages would be useful. Even more useful would be WAV file recordings; but, current hard drive capacities aren't quite there yet; it would use about 1.4 TB per year for a 12 kHz sound card sample rate.

Fortunately, Radio-SkyPipe can record the original sound card averages and the calibrated data at the same time. Taking for example the two Jove receivers on the AJ4CO TFD Array, we simply configure RSP's channels as follows under the Options → Data Source menu item.

Channel 1: Sound Card Left (RCP Jove receiver sound card averaged data)

Channel 2: Sound Card Right (LCP Jove receiver sound card averaged data)

Channel 3: Equation (calibrated T_{ant} RCP)

Channel 4: Equation (calibrated T_{ant} LCP)

Channel 5: Equation (difference between RCP and LCP in terms of dB)

The RSP Data Source dialog is as shown in Fig 24.

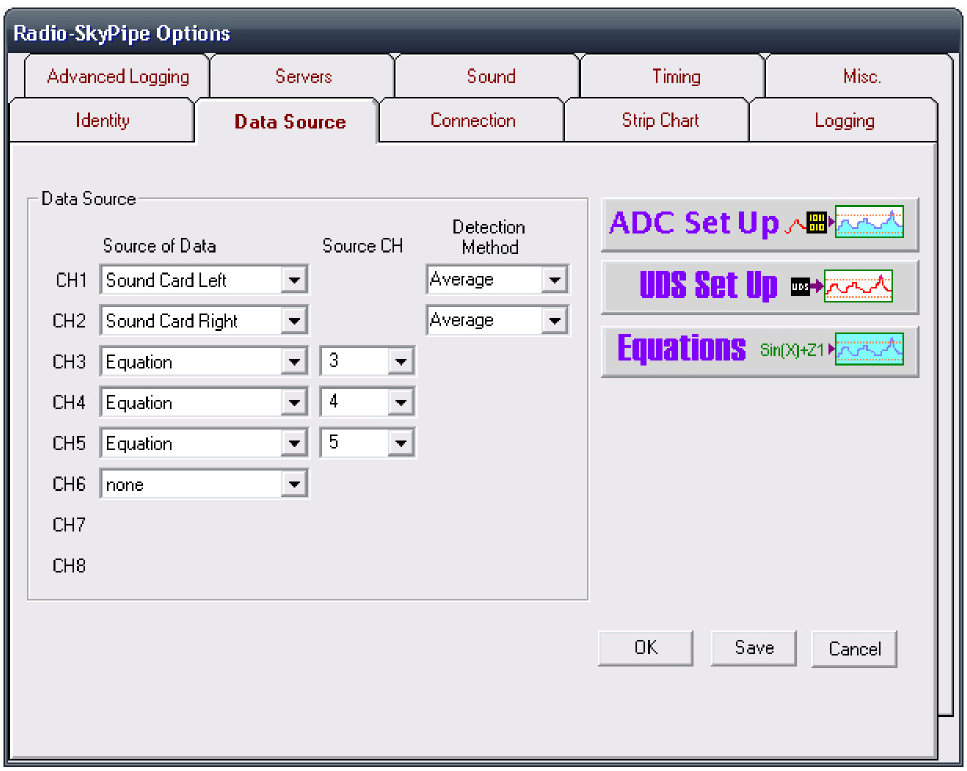


Figure 24 – RSP Data Source configuration dialog.

In the two calibration equations, we substitute Z1 and Z2 for X. This allows the equations in channels 3 and 4 to reference the data in channels 1 (Z1) and 2 (Z2). The three equations are as follows.

For RSP Channel 3 (TFD Array RCP):

```
A=0.00341348*(Z1^2.0111827);B=log10(A);C=-1.1237*B^4+5.7072*B^3-7.2
432*B^2-4.0642*B+9.4958;IF{[A]<[134.77]};THEN{A*10^(-C/10)};ELSE{A}
```

For RSP Channel 4 (TFD Array LCP):

```
A=0.00361555*(Z2^2.0158154);B=log10(A);C=3.6328*B^4-29.587*B^3+91.0
16*B^2-126.47*B+67.574;IF{[A]<[129.65]};THEN{A*10^(-C/10)};ELSE{A}
```

For RSP Channel 5 (dB difference between calibrated T_{ant} in RCP and LCP):

```
10*log10(Z3/Z4)
```

The RSP Equations dialog is as shown in Fig 25.

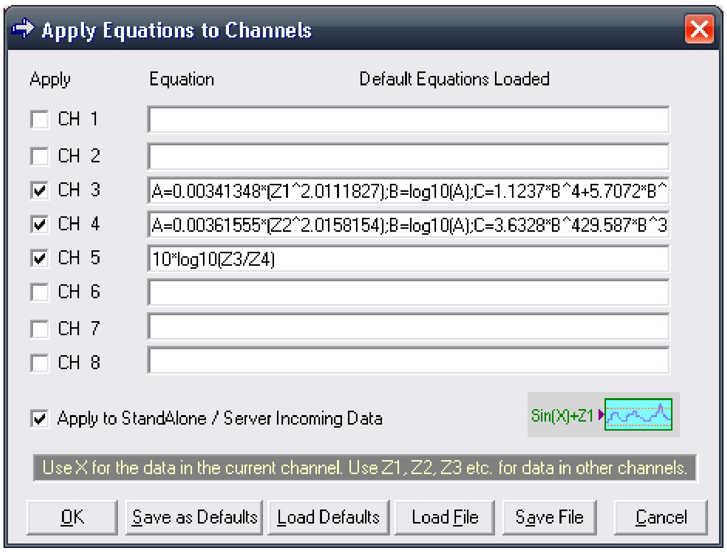


Figure 25 – Final version of the RSP equations for the TFD Array Jove receivers.

Conclusions

The examples discussed in this article show only a few of the many ways that RSP can be calibrated. The equation functionality built into RSP affords the user nearly infinite control. For example, we could have fit a polynomial calibration equation to data using the Power detection method instead of the Average detection method. Or we could have used the Calibration Wizard and then applied a polynomial in the log domain to fine-tune the Cal Wizard’s calibration.

Or we could have used some other pair of functions instead of an exponential followed by a polynomial. If we knew the antenna’s beam characteristics with great accuracy (we usually do not) and the source position within the beam, we could add further equations to calibrate the strip chart in terms of flux density (Janskys).

There are many ways to get the job done and not all have been explored. The Calibration Wizard is excellent for a quick calibration that is accurate over the telescope’s dynamic range – that is, bounded by insensitivity at the low end and compression at the high end. Deriving separate calibration equations – or adding equations to the Cal Wizard’s results – is more befitting those who like to tinker under the hood to get as much accuracy and dynamic range out of the radio telescope as possible.

We note that no matter what method is used, the practical limit of accuracy in T_{ant} for the instruments discussed is around ±0.25 dB. Our attempts to obtain accuracy better than that have all met with failure and are henceforth left to those who are made of sterner stuff.

Last Word – or How I Learned to Stop Worrying and Love the Automatic Calibrator

Calibrations don't last forever. It is important to run periodic step calibrations if a radio telescope's amplitude data are to be of much use. That does not mean running the Cal Wizard every day or deriving calibration equations every day. It does mean that the calibration levels – such as those from an automatic step calibrator – should be present at least daily in the recorded data. This enables a researcher looking at the data to compare the observation of interest with a step calibration not too far removed in time.

References

Flagg, Amplitude Calibration - Measuring Antenna Temperature, **http://www.radiojove.org/SUG/Pubs/Amplitude Calibration - Measuring Antenna Temperature, Flagg (Jove, 2012).pdf**

Flagg, RF 2080 Operating Manual, **https://radiojove.gsfc.nasa.gov/telescope/RF%202080%20Manual.doc**

Sky, Radio-SkyPipe Detection Methods, **http://www.radiosky.com/skypipehelp/V2/sound_card_detection_methods.html**

Sky, Radio-SkyPipe Online Help, **http://www.radiosky.com/skypipehelp/V2/skypipehelpindex.html**

Sky, Writing Equations for SkyPipe, **http://www.radiosky.com/skypipehelp/V2/writingequations.html**

Typinski, AJ4CO Observatory Automatic Calibrator User Manual, **http://www.typnet.net/AJ4CO/Publications/Automatic Calibrator User Manual 2017 06 17.pdf**

Typinski, AJ4CO Observatory Station Description, **http://www.typnet.net/AJ4CO/Publications/AJ4CO Observatory - Station Description 2017 06 24.pdf**